

# THE BERS EMBEDDING OF ONE-DIMENSIONAL TEICHMÜLLER SPACES

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In this poster, we explain how to compute the monodromy representation of a Fuchsian group associated to a given (bounded) projective structure on a marked once-punctured torus or four-times punctured sphere, say  $X$ . The calculation can essentially be reduced to solving (numerically) the homogeneous linear ordinary differential equation

$$2y'' + \left\{ \frac{1}{2z^2(z-1)^2} + \frac{1}{2(z-\lambda)^2} + \frac{t+c(\lambda)}{z(z-1)(z-\lambda)} \right\} y = 0,$$

where  $\lambda$  is a point in  $\mathbb{C} \setminus \{0, 1\}$  such that  $X$  and the domain  $Y = \mathbb{C} \setminus \{0, 1, \lambda\}$  are commensurable with respect to coverings, and  $c(\lambda)$  is a constant which is often called the accessory parameter of  $Y$ . (This method is a modification of that of R. M. Porter and L. Keen developed around 1980.)

Using this, we exhibit pictures of the Bers embedded Teichmüller space of  $X$ , and furthermore, pictures of the interior of the discreteness locus in the space of bounded projective structures on  $X$ . Note that the component of the interior of the discreteness locus containing the “origin” is nothing but the (Bers embedded) Teichmüller space of  $X$  by a result of H. Shiga. A component of the interior of the discreteness locus other than the Teichmüller space of  $X$  is called *exotic*. We will see a mysterious configuration of the exotic components.

We also provide computer graphics of the bending loci corresponding to simple closed curves on  $X$ . As applications, numerically, we can calculate the accessory parameter, and we can give the inner (and possibly, outer) radius of the Bers embedding of the Teichmüller space of the surface  $\mathbb{C} \setminus \{0, 1, -1\}$ . Also, we can give the Maskit coordinate of a given Teichmüller parameter in a numerical way.

**Information:** You can see related pictures in the following website:

<http://www.kusm.kyoto-u.ac.jp/complex/Bers/>

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