In this talk, we present applications of holomorphic motions to the quasiconformal extension of univalent functions. Compare the following two results.

**Theorem A** (Krzyż [2]). Let $\omega$ be an analytic function on the unit disk $\Delta$ with $|\omega'(z)| \leq k$, where $0 \leq k < 1$ is a constant. Then the function $f(z) = z + \omega(1/z)$ on the outside of $\Delta$ can be extended to a $k$-quasiconformal automorphism of the Riemann sphere.

**Theorem B** (Fait, Krzyż and Zygmunt [1, Theorem 2']). Let $\omega$ be an analytic function on the unit disk $\Delta$ with $|\omega'(z)| \leq k$, where $0 \leq k < 1$ is a constant. Then the function $f(z) = z + \omega(z)$ on $\Delta$ can be extended to a $k'$-quasiconformal automorphism of the Riemann sphere, where $k' = k/(2 - k)$. In fact, this can be easily obtained as a corollary of the following result, which is one of our results presented here.

**Theorem.** Let $k$ be a constant in $[0,1)$. For a normalized analytic function $f(z) = z + a_2 z^2 + \ldots$ on the unit disk $\Delta$, let $p(z)$ represent one of the quantities $zf'(z)/f(z)$, $1 + zf''(z)/f'(z)$ or $f''(z)$. If $p(\Delta) \subset \{w \in \mathbb{C}; |w - (1 + k^2)/(1 - k^2)| \leq 2k/(1 - k^2)\}$, the function $f$ can be extended to a $k'$-quasiconformal automorphism of the Riemann sphere.

In the case that $p(z) = zf'(z)/f(z)$ the above result is best possible as the function $p(z) = (1 + k z^2)/(1 - k z^2)$ indicates.

The above theorem can be shown by means of the optimal $\lambda$-lemma first proved by Słodkowski [3]. We will present a general principle which enables us to deduce quasiconformal extension criteria from only univalence criteria.

**References**


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